

Introduction to **DIGITAL**  
**SIGNAL**  
**PROCESSING** and  
**FILTER DESIGN**

**B. A. SHENOI**



# CONTENTS

<b>Preface</b>	<b>xi</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Introduction	1
1.2 Applications of DSP	1
1.3 Discrete-Time Signals	3
1.3.1 Modeling and Properties of Discrete-Time Signals	8
1.3.2 Unit Pulse Function	9
1.3.3 Constant Sequence	10
1.3.4 Unit Step Function	10
1.3.5 Real Exponential Function	12
1.3.6 Complex Exponential Function	12
1.3.7 Properties of $\cos(\omega_0 n)$	14
1.4 History of Filter Design	19
1.5 Analog and Digital Signal Processing	23
1.5.1 Operation of a Mobile Phone Network	25
1.6 Summary	28
Problems	29
References	30
<b>2 Time-Domain Analysis and <math>z</math> Transform</b>	<b>32</b>
2.1 A Linear, Time-Invariant System	32
2.1.1 Models of the Discrete-Time System	33
2.1.2 Recursive Algorithm	36
2.1.3 Convolution Sum	38
2.2 $z$ Transform Theory	41
2.2.1 Definition	41
2.2.2 Zero Input and Zero State Response	49

2.2.3	Linearity of the System	50
2.2.4	Time-Invariant System	50
2.3	Using $z$ Transform to Solve Difference Equations	51
2.3.1	More Applications of $z$ Transform	56
2.3.2	Natural Response and Forced Response	58
2.4	Solving Difference Equations Using the Classical Method	59
2.4.1	Transient Response and Steady-State Response	63
2.5	$z$ Transform Method Revisited	64
2.6	Convolution Revisited	65
2.7	A Model from Other Models	70
2.7.1	Review of Model Generation	72
2.8	Stability	77
2.8.1	Jury–Marden Test	78
2.9	Solution Using MATLAB Functions	81
2.10	Summary	93
	Problems	94
	References	110
<b>3</b>	<b>Frequency-Domain Analysis</b>	<b>112</b>
3.1	Introduction	112
3.2	Theory of Sampling	113
3.2.1	Sampling of Bandpass Signals	120
3.3	DTFT and IDTFT	122
3.3.1	Time-Domain Analysis of Noncausal Inputs	125
3.3.2	Time-Shifting Property	127
3.3.3	Frequency-Shifting Property	127
3.3.4	Time Reversal Property	128
3.4	DTFT of Unit Step Sequence	138
3.4.1	Differentiation Property	139
3.4.2	Multiplication Property	142
3.4.3	Conjugation Property	145
3.4.4	Symmetry Property	145
3.5	Use of MATLAB to Compute DTFT	147
3.6	DTFS and DFT	154
3.6.1	Introduction	154



3.6.2	Discrete-Time Fourier Series	156
3.6.3	Discrete Fourier Transform	159
3.6.4	Reconstruction of DTFT from DFT	160
3.6.5	Properties of DTFS and DFT	161
3.7	Fast Fourier Transform	170
3.8	Use of MATLAB to Compute DFT and IDFT	172
3.9	Summary	177
	Problems	178
	References	185
<b>4</b>	<b>Infinite Impulse Response Filters</b>	<b>186</b>
4.1	Introduction	186
4.2	Magnitude Approximation of Analog Filters	189
4.2.1	Maximally Flat and Butterworth Approximation	191
4.2.2	Design Theory of Butterworth Lowpass Filters	194
4.2.3	Chebyshev I Approximation	202
4.2.4	Properties of Chebyshev Polynomials	202
4.2.5	Design Theory of Chebyshev I Lowpass Filters	204
4.2.6	Chebyshev II Approximation	208
4.2.7	Design of Chebyshev II Lowpass Filters	210
4.2.8	Elliptic Function Approximation	212
4.3	Analog Frequency Transformations	212
4.3.1	Highpass Filter	212
4.3.2	Bandpass Filter	213
4.3.3	Bandstop Filter	216
4.4	Digital Filters	219
4.5	Impulse-Invariant Transformation	219
4.6	Bilinear Transformation	221
4.7	Digital Spectral Transformation	226
4.8	Allpass Filters	230
4.9	IIR Filter Design Using MATLAB	231
4.10	Yule-Walker Approximation	238
4.11	Summary	240
	Problems	240
	References	247



<b>5</b>	<b>Finite Impulse Response Filters</b>	<b>249</b>
5.1	Introduction	249
5.1.1	Notations	250
5.2	Linear Phase Fir Filters	251
5.2.1	Properties of Linear Phase FIR Filters	256
5.3	Fourier Series Method Modified by Windows	261
5.3.1	Gibbs Phenomenon	263
5.3.2	Use of Window Functions	266
5.3.3	FIR Filter Design Procedures	268
5.4	Design of Windowed FIR Filters Using MATLAB	273
5.4.1	Estimation of Filter Order	273
5.4.2	Design of the FIR Filter	275
5.5	Equiripple Linear Phase FIR Filters	280
5.6	Design of Equiripple FIR Filters Using MATLAB	285
5.6.1	Use of MATLAB Program to Design Equiripple FIR Filters	285
5.7	Frequency Sampling Method	289
5.8	Summary	292
	Problems	294
	References	301
<b>6</b>	<b>Filter Realizations</b>	<b>303</b>
6.1	Introduction	303
6.2	FIR Filter Realizations	305
6.2.1	Lattice Structure for FIR Filters	309
6.2.2	Linear Phase FIR Filter Realizations	310
6.3	IIR Filter Realizations	312
6.4	Allpass Filters in Parallel	320
6.4.1	Design Procedure	325
6.4.2	Lattice-Ladder Realization	326
6.5	Realization of FIR and IIR Filters Using MATLAB	327
6.5.1	MATLAB Program Used to Find Allpass Filters in Parallel	334
6.6	Summary	346

Problems	347
References	353
<b>7 Quantized Filter Analysis</b>	<b>354</b>
7.1 Introduction	354
7.2 Filter Design–Analysis Tool	355
7.3 Quantized Filter Analysis	360
7.4 Binary Numbers and Arithmetic	360
7.5 Quantization Analysis of IIR Filters	367
7.6 Quantization Analysis of FIR Filters	375
7.7 Summary	379
Problems	379
References	379
<b>8 Hardware Design Using DSP Chips</b>	<b>381</b>
8.1 Introduction	381
8.2 Simulink and Real-Time Workshop	381
8.3 Design Preliminaries	383
8.4 Code Generation	385
8.5 Code Composer Studio	386
8.6 Simulator and Emulator	388
8.6.1 Embedded Target with Real-Time Workshop	389
8.7 Conclusion	389
References	390
<b>9 MATLAB Primer</b>	<b>391</b>
9.1 Introduction	391
9.1.1 Vectors, Arrays, and Matrices	392
9.1.2 Matrix Operations	393
9.1.3 Scalar Operations	398
9.1.4 Drawing Plots	400
9.1.5 MATLAB Functions	400
9.1.6 Numerical Format	401



9.1.7	Control Flow	402
9.1.8	Edit Window and M-file	403
9.2	Signal Processing Toolbox	405
9.2.1	List of Functions in Signal Processing Toolbox	406
	References	414
	<b>Index</b>	<b>415</b>



## **PREFACE**

---

This preface is addressed to instructors as well as students at the junior–senior level for the following reasons. I have been teaching courses on digital signal processing, including its applications and digital filter design, at the undergraduate and the graduate levels for more than 25 years. One common complaint I have heard from undergraduate students in recent years is that there are not enough numerical problems worked out in the chapters of the book prescribed for the course. But some of the very well known textbooks on digital signal processing have more problems than do a few of the books published in earlier years. However, these books are written for students in the senior and graduate levels, and hence the junior-level students find that there is too much of mathematical theory in these books. They also have concerns about the advanced level of problems found at the end of chapters. I have not found a textbook on digital signal processing that meets these complaints and concerns from junior-level students. So here is a book that I have written to meet the junior students' needs and written with a student-oriented approach, based on many years of teaching courses at the junior level.

*Network Analysis* is an undergraduate textbook authored by my Ph.D. thesis advisor Professor M. E. Van Valkenburg (published by Prentice-Hall in 1964), which became a world-famous classic, not because it contained an abundance of all topics in network analysis discussed with the rigor and beauty of mathematical theory, but because it helped the students understand the basic ideas in their simplest form when they took the first course on network analysis. I have been highly influenced by that book, while writing this textbook for the first course on digital signal processing that the students take. But I also have had to remember that the generation of undergraduate students is different; the curriculum and the topic of digital signal processing is also different. This textbook does not contain many of the topics that are found in the senior–graduate-level textbooks mentioned above. One of its main features is that it uses a very large number of numerical problems as well as problems using functions from MATLAB® (MATLAB is a registered trademark of The MathWorks, Inc.) and Signal Processing Toolbox, worked out in every chapter, in order to highlight the fundamental concepts. These problems are solved as examples after the theory is discussed or are worked out first and the theory is then presented. Either way, the thrust of the approach is that the students should understand the basic ideas, using the worked, out problems as an instrument to achieve that goal. In some cases, the presentation is more informal than in other cases. The students will find statements beginning with “Note that. . .,” “Remember. . .,” or “It is pointed out,” and so on; they are meant



to emphasize the important concepts and the results stated in those sentences. Many of the important results are mentioned more than once or summarized in order to emphasize their significance.

The other attractive feature of this book is that all the problems given at the end of the chapters are problems that can be solved by using only the material discussed in the chapters, so that students would feel confident that they have an understanding of the material covered in the course when they succeed in solving the problems. Because of such considerations mentioned above, the author claims that the book is written with a student-oriented approach. Yet, the students should know that the ability to understand the solution to the problems is important but understanding the theory behind them is far more important.

The following paragraphs are addressed to the instructors teaching a junior-level course on digital signal processing. The first seven chapters cover well-defined topics: (1) an introduction, (2) time-domain analysis and  $z$ -transform, (3) frequency-domain analysis, (4) infinite impulse response filters, (5) finite impulse response filters, (6) realization of structures, and (7) quantization filter analysis. Chapter 8 discusses hardware design, and Chapter 9 covers MATLAB. The book treats the mainstream topics in digital signal processing with a well-defined focus on the fundamental concepts.

Most of the senior-graduate-level textbooks treat the theory of finite wordlength in great detail, but the students get no help in analyzing the effect of finite wordlength on the frequency response of a filter or designing a filter that meets a set of frequency response specifications with a given wordlength and quantization format. In Chapter 7, we discuss the use of a MATLAB tool known as the "FDA Tool" to thoroughly investigate the effect of finite wordlength and different formats of quantization. This is another attractive feature of the textbook, and the material included in this chapter is not found in any other textbook published so far.

When the students have taken a course on digital signal processing, and join an industry that designs digital signal processing (DSP) systems using commercially available DSP chips, they have very little guidance on what they need to learn. It is with that concern that additional material in Chapter 8 has been added, leading them to the material that they have to learn in order to succeed in their professional development. It is very brief but important material presented to guide them in the right direction. The textbooks that are written on DSP hardly provide any guidance on this matter, although there are quite a few books on the hardware implementation of digital systems using commercially available DSP chips. Only a few schools offer laboratory-oriented courses on the design and testing of digital systems using such chips. Even the minimal amount of information in Chapter 8 is not found in any other textbook that contains "digital signal processing" in its title. However, Chapter 8 is not an exhaustive treatment of hardware implementation but only as an introduction to what the students have to learn when they begin a career in the industry.

Chapter 1 is devoted to discrete-time signals. It describes some applications of digital signal processing and defines and, suggests several ways of describing discrete-time signals. Examples of a few discrete-time signals and some basic



operations applied with them is followed by their properties. In particular, the properties of complex exponential and sinusoidal discrete-time signals are described. A brief history of analog and digital filter design is given. Then the advantages of digital signal processing over continuous-time (analog) signal processing is discussed in this chapter.

Chapter 2 is devoted to discrete-time systems. Several ways of modeling them and four methods for obtaining the response of discrete-time systems when excited by discrete-time signals are discussed in detail. The four methods are (1) recursive algorithm, (2) convolution sum, (3) classical method, and (4)  $z$ -transform method to find the total response in the time domain. The use of  $z$ -transform theory to find the zero state response, zero input response, natural and forced responses, and transient and steady-state responses is discussed in great detail and illustrated with many numerical examples as well as the application of MATLAB functions. Properties of discrete-time systems, unit pulse response and transfer functions, stability theory, and the Jury–Marden test are treated in this chapter. The amount of material on the time-domain analysis of discrete-time systems is a lot more than that included in many other textbooks.

Chapter 3 concentrates on frequency-domain analysis. Derivation of sampling theorem is followed by the derivation of the discrete-time Fourier transform (DTFT) along with its importance in filter design. Several properties of DTFT and examples of deriving the DTFT of typical discrete-time signals are included with many numerical examples worked out to explain them. A large number of problems solved by MATLAB functions are also added. This chapter devoted to frequency-domain analysis is very different from those found in other textbooks in many respects.

The design of infinite impulse response (IIR) filters is the main topic of Chapter 4. The theory of approximation of analog filter functions, design of analog filters that approximate specified frequency response, the use of impulse-invariant transformation, and bilinear transformation are discussed in this chapter. Plenty of numerical examples are worked out, and the use of MATLAB functions to design many more filters are included, to provide a hands-on experience to the students.

Chapter 5 is concerned with the theory and design of finite impulse response (FIR) filters. Properties of FIR filters with linear phase, and design of such filters by the Fourier series method modified by window functions, is a major part of this chapter. The design of equiripple FIR filters using the Remez exchange algorithm is also discussed in this chapter. Many numerical examples and MATLAB functions are used in this chapter to illustrate the design procedures.

After learning several methods for designing IIR and FIR filters from Chapters 4 and 5, the students need to obtain as many realization structures as possible, to enable them to investigate the effects of finite wordlength on the frequency response of these structures and to select the best structure. In Chapter 6, we describe methods for deriving several structures for realizing FIR filters and IIR filters. The structures for FIR filters describe the direct, cascade, and polyphase forms and the lattice structure along with their transpose forms. The structures for



IIR filters include direct-form and cascade and parallel structures, lattice-ladder structures with autoregressive (AR), moving-average (MA), and allpass structures as special cases, and lattice-coupled allpass structures. Again, this chapter contains a large number of examples worked out numerically and using the functions from MATLAB and Signal Processing Toolbox; the material is more than what is found in many other textbooks.

The effect of finite wordlength on the frequency response of filters realized by the many structures discussed in Chapter 6 is treated in Chapter 7, and the treatment is significantly different from that found in all other textbooks. There is no theoretical analysis of finite wordlength effect in this chapter, because it is beyond the scope of a junior-level course. I have chosen to illustrate the use of a MATLAB tool called the "FDA Tool" for investigating these effects on the different structures, different transfer functions, and different formats for quantizing the values of filter coefficients. The additional choices such as truncation, rounding, saturation, and scaling to find the optimum filter structure, besides the alternative choices for the many structures, transfer functions, and so on, makes this a more powerful tool than the theoretical results. Students would find experience in using this tool far more useful than the theory in practical hardware implementation.

Chapters 1–7 cover the core topics of digital signal processing. Chapter 8, on hardware implementation of digital filters, briefly describes the simulation of digital filters on Simulink<sup>®</sup>, and the generation of C code from Simulink using Real-Time Workshop<sup>®</sup> (Simulink and Real-Time Workshop are registered trademarks of The MathWorks, Inc.), generating assembly language code from the C code, linking the separate sections of the assembly language code to generate an executable object code under the Code Composer Studio from Texas Instruments is outlined. Information on DSP Development Starter kits and simulator and emulator boards is also included. Chapter 9, on MATLAB and Signal Processing Toolbox, concludes the book.

The author suggests that the first three chapters, which discuss the basics of digital signal processing, can be taught at the junior level in one quarter. The prerequisite for taking this course is a junior-level course on linear, continuous-time signals and systems that covers Laplace transform, Fourier transform, and Fourier series in particular. Chapters 4–7, which discuss the design and implementation of digital filters, can be taught in the next quarter or in the senior year as an elective course depending on the curriculum of the department. Instructors must use discretion in choosing the worked-out problems for discussion in the class, noting that the real purpose of these problems is to help the students understand the theory. There are a few topics that are either too advanced for a junior-level course or take too much of class time. Examples of such topics are the derivation of the objective function that is minimized by the Remez exchange algorithm, the formulas for deriving the lattice-ladder realization, and the derivation of the fast Fourier transform algorithm. It is my experience that students are interested only in the use of MATLAB functions that implement these algorithms, and hence I have deleted a theoretical exposition of the last two topics and also a description



of the optimization technique in the Remez exchange algorithm. However, I have included many examples using the MATLAB functions to explain the subject matter.

Solutions to the problems given at the end of chapters can be obtained by the instructors from the Website <http://www.wiley.com/WileyCDA/WileyTitle/productCd-0471464821.html>. They have to access the solutions by clicking "Download the software solutions manual link" displayed on the Webpage. The author plans to add more problems and their solutions, posting them on the Website frequently after the book is published.

As mentioned at the beginning of this preface, the book is written from my own experience in teaching a junior-level course on digital signal processing. I wish to thank Dr. M. D. Srinath, Southern Methodist University, Dallas, for making a thorough review and constructive suggestions to improve the material of this book. I also wish to thank my colleague Dr. A. K. Shaw, Wright State University, Dayton. And I am most grateful to my wife Suman, who has spent hundreds of lonely hours while I was writing this book. Without her patience and support, I would not have even started on this project, let alone complete it. So I dedicate this book to her and also to our family.

B. A. SHENOI

*May 2005*